QUANTUM COMPUTING FROM COLOSSUS TO QUBITS

THE HISTORY, THEORY, AND APPLICATION OF A REVOLUTIONARY SCIENCE

JOHN GRIBBIN

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Foreword

In Search of the Quantum Computer

Over the past seventy-five years, the world has been transformed by what might loosely be called electronic technology – transistors, semiconductor chips, lasers and the like. It is thanks to this technology that we have smartphones, weather-forecasting computers and self-driving cars, to name just a few of the products it has generated. The reason for these huge technical advances is that they exploit some of the 'quantum' properties of components, such as microchips and the electrons that flow through them. But this was only a beginning. Physicists now refer to all of these advances as being based on 'Quantum 1.0' technology, and are now developing 'Quantum 2.0' technology, which will take us as far beyond the technology of the past seventy-five years as that technology took us beyond the steam-powered technology of the nineteenth century.

This is because the new leap forward is based upon the ability to make use of the strangest quantum properties of things like electrons – their ability to exist in a mixture of states, called a superposition, and the capacity for one entity to be influenced instantaneously by what happens to its counterpart far away, called entanglement. You may have come across these terms in books and articles describing the weirdness of the quantum world, talking about Schrödinger's Cat and Spooky Action at a Distance, and dismissed them as the crazy dreams of the more wild-eyed kind of scientist. But they are real, and they can now be manipulated in a variety of ways, most notably in the form of a completely different kind of computer, which is the theme of this book.

Perhaps the best way to bring home the impact Quantum 2.0 technology is already having on our world is to follow the money. I am writing this in the summer of 2022, so the latest figures I have are for late 2021. At an event billed as the UK's National Quantum Technologies Showcase, held in London, grants totalling more than £300 million for the development of quantum technologies were announced, and a joint declaration of intent between the UK and the USA was signed with the stated intention to 'boost collaboration on quantum science and technologies'. There were by then forty-one quantum technology companies operating in the UK, and in a highly significant development Honeywell bought up the Cambridge Quantum Computing company. The new joint venture, 'Quantinium', kickstarted with an investment of just under $300 million from Honeywell, has headquarters in both Colorado, USA, and Cambridge, England, and employs some four hundred people. When the big boys start buying out the start-ups, you know something is happening. Alongside quantum computing, dramatic advances are being made in quantum encryption, techniques to generate uncrackable codes, a quantum internet, and applications such as imaging. Quantum technology has arrived and is with us today.

My focus here, however, is on the key quantum technology, the development of quantum computing. The first edition of this book was published in 2013, and did not go into details about the latest technical developments in the field. That field is developing so fast that any attempt to be up to date in that sense is doomed anyway – there will be new announcements of new breakthroughs in the months between my writing these words and your reading them. But that is not what the book is about. I aim here to explain the fundamental nature of quantum computing, set in the context of the development of electronic computers in the twentieth century. Armed with this, you will be better equipped to understand what is going on as we move into a world dominated by Quantum 2.0 technology, and won't be fazed by references to things like entanglement and superposition. But I cannot resist giving you a hint of how things have developed since 2013, particularly with the team at my own base, the University of Sussex. If some of this news goes a little over your head, don't worry – you can come back to it after you have read the book.

The power of quantum computers rests upon the ability of single quantum entities (such as, but not exclusively, electrons) to store information as what are called qubits, by analogy with the 'bits' of a conventional computer. The technology of quantum computing requires the manipulation of those qubits, and one of the key problems is to find a way to do this without introducing errors. A string of qubits makes up an instruction in computer code, just as a string of letters and spaces makes up this sentence. Errors in the code are like typos in the sentence, and can scramble up the message. Strings of qubits have to be checked for errors (using other strings of code), and they have to last long enough in a particular quantum state for something useful to be done with them. One way of achieving this is to place ‘dots’ of so-called donor atoms (such as phosphorus) in a material such as silicon and tweak them with electromagnetic fields. Recent studies have shown that in such circumstances the state of an electron associated with the phosphorus atom can be preserved for ten seconds, while the quantum state of an atomic nucleus can be preserved for a few hours. But there are practical problems with this kind of setup, not least that the donor atoms have to be very close together (less than 15 nanometres apart) for the necessary quantum interactions between the qubits to take place. This technique is still at the laboratory stage of development.

The Sussex team, using a different technology, are among the researchers moving out of the lab and into practical development of a quantum computer. They use the trapped-ion technique described in chapter 6, and have now formed a company, Universal Quantum, which is taking that technology into the commercial world.

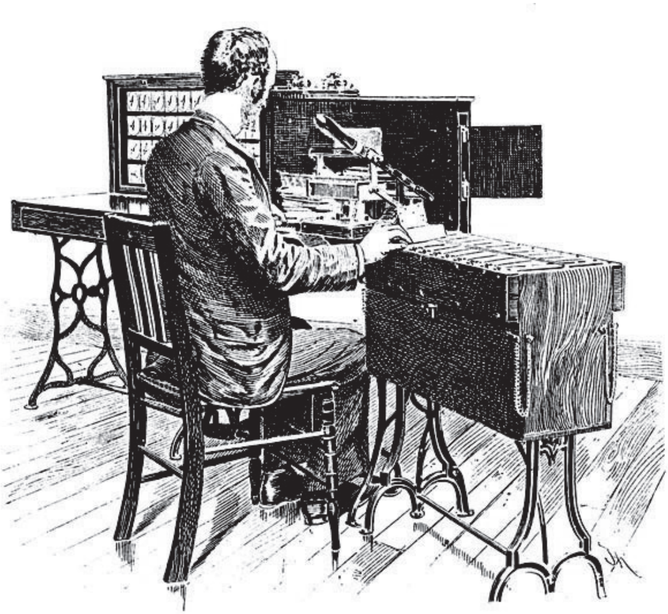
Universal Quantum is part of a consortium also involving a group at Imperial College London and Rolls-Royce. They are working on the technology of error correction (fixing typos in strings of qubits), which has been described as the Holy Grail of quantum computing. A short chain of qubits in a working program is likely to require hundreds or maybe thousands more qubits in the error-correcting code, and unlike the competing technologies the trapped-ion technique offers a practical way of scaling up the technology to make this a feasible proposition. The quantum dot technique works on a small scale, but is hard to scale up. And the involvement of

Rolls-Royce highlights the widespread applications of the Quantum 2.0 revolution. The project is using as a test case a computer model of the processes that go on in jet engines, involving complex problems of fluid dynamics that cannot be solved using conventional computers. A quantum computer that can solve this kind of problem might occupy an area bigger than a tennis court – but the first conventional computers were just as big, and soon led to laptops and smartphones. As I mentioned in 2013, at that time Professor Winfried Hensinger at Sussex was hoping to have a working version of the trapped-ion computer by the mid-2020s. It doesn't look as if that forecast will be far wrong.

The timing is intriguing. It was as recently as 1925 that Werner Heisenberg and Erwin Schrödinger independently came up with a working theory of quantum mechanics. To mark the centenary of that double breakthrough, physicists around the world are planning a series of commemorative events. There is a proposal to make 2025 the UNESCO International Year of Quantum Science and Technology. The first step is to have the proposal discussed at a meeting of the UNESCO executive board, before it goes to the UN General Assembly a year later for formal approval. It would be fitting if a working quantum computer came online in the centenary year of the foundation of quantum mechanics.

John Gribbin
August 2022

PART ONE



Card-based census counting machine, 1890.
One of the forerunners of the computer.

Chapter One

Turing and the Machine

If necessity is the mother of invention, the computer had two mothers – cryptography and the hydrogen bomb. But there was only one father: Alan Mathison Turing.

A child of empire

Turing was conceived in India, where his father, Julius, was a member of the Indian Civil Service helping to administer this jewel in the crown of the British Empire; but he was born, on 23 June 1912, in Maida Vale, London, when his parents were on home leave. He already had a brother, John, born in India on 1 September 1908. When Julius returned to India their mother, Sara,[1] stayed in England with the two boys, but only until September 1913, when she rejoined her husband and left the children in the care of a retired army colonel and his wife, who lived at St Leonards-on-Sea in Sussex. There was a nanny who looked after the two boys and the colonel's four daughters, together with another boy whose parents were overseas, and later three cousins of Alan and John. Their mother returned for the summer of 1915, staying with the boys in rented rooms in St Leonards, and both parents came to England in the spring of 1916 – the first time that Alan really had an opportunity to get to know his father. At the end of this leave, in August, Julius Turing returned to India for his next three years' tour of duty. John had already been sent away to school at Hazelhurst, in Kent; Alan, having been just one of a motley group of children, now became in effect the only child of a single parent, who took him almost everywhere with her, including to the High Anglican church she attended (which he hated) and to art classes (she was an accomplished watercolourist), where he was the darling of the female students.

Alan was remembered as a bright, untidy child with a predilection for inventing his own words, such as 'quockling' to describe the sound of seagulls and 'greasicle' for a guttering candle. It was impossible to pull the wool over his eyes – when his nanny tried to let him win a game they were playing by making poor moves, he saw through the subterfuge and was infuriated; when his mother was reading him a story and left a dull passage out, he yelled: 'You spoil the whole thing.'[2] Nor was he ever in any doubt about the accuracy of his own world-view: he *knew*, for example, that the fruit which tempted Eve in the Garden of Eden was a plum. But he never could tell left from right, and marked his left thumb with a red spot so that he would know which was which.

Having taught himself to read (from a book appropriately called *Reading Without Tears*), Alan first encountered formal education at the age of six, when his mother sent him to a local day school to learn Latin. This failed to stir his interest, but highlighted his great difficulty with the physical process of writing, especially with the ink pens in use at the time. His work was always a mess of scratchy scribbles, crossings-out and blots, reminiscent of nothing so much as the spoof handiwork of Nigel Molesworth in the stories by Geoffrey Willans and Ronald Searle.

Alan's next meeting with his father came in 1919, when Julius's leave included a holiday in Scotland: here the seven-year-old boy impressed his family on a picnic by tracking the flights of wild bees to their intersection to find honey. But in December both parents sailed for India, and Alan returned to the colonel's house in St Leonards while John went back to school in Hazelhurst. The next two years saw a change in Alan. When his mother next returned, in 1921, she found that the vivacious and friendly boy she had left in England had become 'unsociable and dreamy', while his education had been so neglected that at nearly nine he had not learned how to do long division. She took him away for a holiday in Brittany, and then to London, where she taught him long division herself. She later recalled that when he had been shown how to find the square root of a number, he worked out for himself how to find the cube root.

At the beginning of 1922, it was time for Alan to follow his brother John to Hazelhurst, a small school for thirty-six boys aged from nine to thirteen, with just three teachers and a matron who looked after the boys. The brothers were together at Hazelhurst for only one term before John left at Easter for Marlborough College and the public-school education for which 'prep' schools such as Hazelhurst were preparing their boys. The same year, Alan was given a book called *Natural Wonders Every Child Should Know*, by Edwin

Brewster. This first encounter with science made a deep impression on him, especially the way the author likened the workings of the body, even the brain, to a machine. He was less impressed by the sporting activities that young English gentlemen of the time were expected to enjoy (or at least endure), and later claimed that he had learned to run fast (he became a very good long-distance runner as an adult) in order to keep away from the ball during hockey. He was also disturbed by the imprecision of some of his teachers, and wrote to John that one of them 'gave a quite false impression of what is meant by x'. His concern was not for himself, but that the other boys might be misled.

The summer of 1922 brought the return of Alan's father on leave once more, and another happy family holiday in Scotland. But in September his parents left him back at Hazelhurst, departing down the drive of the school with Sara biting her lip as she watched her son running futilely after the taxi, trying to catch up with them. Bored by school, Alan achieved nothing spectacular in the way of marks, but loved inventing things and developed a passion for chemistry – which was purely a hobby: God forbid that a prep school like Hazelhurst should have anything to do with science. Science was almost as conspicuous by its absence at most public schools, so when in the autumn of 1925 Alan surprised everyone by doing well in the Common Entrance examination that was a prerequisite to the transition, his future presented his parents with something of a conundrum. John made an impassioned plea to their parents not to send his unusual younger brother to Marlborough, which 'will crush the life out of him', and Sara Turing worried that her son might 'become a mere intellectual crank' if he failed to adapt to public school life. The puzzle of what to do with him was solved by a friend of hers who was married to a science master at Sherborne, a school in Dorset established back in 1550 and brought into the modern public school system in 1869. The friend assured Sara that this would be a safe haven for her boy, and Alan started there in 1926.

Sherborne

He was due to arrive for the start of the summer term, on 3 May, from Brittany, where his parents were living to avoid paying British income tax. On the ferry to Southampton, Alan learned that there would be no trains, because of the general strike; totally unfazed, and still a month short of his fourteenth birthday, he cycled the 60 miles to Sherborne, staying overnight at Blandford Forum. This initiative was sufficiently unusual to merit a comment in the *Western Gazette* on 14 May. The same initiative and independence were shown a little later when Alan worked out for himself the formula known as 'Gregory's series' for the inverse tangent, unaware that it had been discovered in 1668 by the Scottish mathematician James Gregory (inventor of a kind of telescope that also bears his name), and even earlier by the Indian mathematician Madhava.

Alan soon settled into his old habit of largely ignoring lessons that he found boring, then doing well in examinations, while continuing his private chemistry experiments and amusing himself with advanced mathematics. At Sherborne, grades depended on a combination of continuous assessment and examinations, each marked separately but with a final combined mark. On one occasion, Alan came twenty-second out of twenty-three for his term's work, first in the examinations, and third overall. His headmaster did not approve of such behaviour, and wrote to Alan's father: 'I hope he will not fall between two stools. If he is to stay at a Public School, he must aim at becoming educated. If he is to be solely a Scientific Specialist, he is wasting his time at a Public School.' But Alan escaped expulsion, and was rather grudgingly allowed to take the School Certificate examination, which had to be passed before he could move on to the sixth form at the beginning of 1929. His immediate future after school, however, was decided as much by love as by logic.

As in all public schools, filled with teenage boys with no other outlet for their burgeoning sexuality, there were inevitably liaisons between older and younger pupils, no matter how much such relationships might be officially frowned upon. It was in this environment that Alan realized that he was homosexual, although there is no record of his having any physical relationships with other boys at school. He did, though, develop something more than a crush on a boy a year ahead of him at school, Christopher Morcom.

The attraction was as much mental as physical (indeed, from Morcom's side it was all mental). Morcom was another mathematician, with whom Alan could discuss science, including Einstein's general theory of relativity, astronomy, and quantum mechanics. He was a star pupil who worked hard at school and achieved high grades in examinations, giving Alan, used to taking it easy and relying on brilliance to get him through, something to strive to emulate. The examination they were both working for, the Higher School Certificate (or just 'Higher'), was a prerequisite to moving on to university. In the mathematics paper they sat, Alan scored a respectable 1,033 marks; but Morcom, the elder by a year, scored 1,436.

In 1929, Morcom was to take the examination for a scholarship at Trinity College, Cambridge. He was eighteen, and expected to pass. Alan was desperate not to see his friend go on to Cambridge without him. He decided to take the scholarship examination at the same time, even though he was still only seventeen and Trinity was the top college in Britain (arguably, in the world) for the study of maths and science, with a correspondingly high admission standard. The examinations were held over a week in Cambridge, giving the two Shirburnians a chance to live the life of undergraduates, and to meet new people, including Maurice Pryce, another candidate, whom Alan would meet again when their paths crossed in Princeton a few years later.

The outcome was as Alan had feared. Morcom passed, gaining a scholarship to Trinity that gave him sufficient income to live on as an undergraduate. Alan did not, and faced a separation of at least a year from his first love. But the separation became permanent when Morcom died, of tuberculosis, on 13 February 1930. Alan wrote to his own mother: 'I feel that I shall meet Morcom again somewhere and that there will be some work for us to do together . . . Now that I am left to do it alone I must not let him down.' And in the spirit of doing the work that they might have done together, or that Morcom might have done alone, and 'not letting him down', Alan tried once again for Cambridge in 1930. Once again, he failed to obtain a Trinity scholarship; but this time he was offered a scholarship worth £80 a year at his second choice of college, King's. He started there in 1931, when he was nineteen.

Cambridge . . .

Turing managed the unusual feat of joining in both the sporting life (as a runner and rower) and the academic life in Cambridge, while never quite fitting in anywhere socially. He also enjoyed at least one homosexual relationship, with another maths student, James Atkins. But it is his mathematical work that is important here. Turing's parting gift from Sherborne, in the form of a prize for his work, had been the book *Mathematical Foundations of Quantum Mechanics*, by the Hungarian-born mathematician John von Neumann, who would soon play a personal part in Turing's story.[3] In an echo of his early days at Sherborne, not long after he arrived in Cambridge Turing independently came up with a theorem previously (unbeknown to him) proved by the Polish mathematician Wacław Sierpiński; when Sierpiński's priority was pointed out to him, he was delighted to find that his proof was simpler than that of the Pole. Polish mathematicians would also soon loom large in Turing's life.

In the early 1930s, the structure of the mathematics course in Cambridge was changing. Everybody who entered in 1931 (eighty-five students in all) took two key examinations, Part I at the end of the first year and Part II at the end of the third year. So-called 'Schedule A' students left it at that, which was sufficient to gain them their degrees. But 'Schedule B' students, including Turing, took a further, more advanced, examination, also at the end of their third year. For the intake which followed Turing's year, the extra examination was taken after a further (fourth) year of study, as it has been ever since: it became known as Part III, and is now roughly equivalent to a Master's degree from other universities.

This peculiarity of the Cambridge system partly explains why Turing never worked for a PhD in Cambridge. Having passed his examinations with flying colours, he was offered a studentship worth £200 which enabled him to stay on at Cambridge for a year to write a dissertation with which he hoped to impress the authorities sufficiently to be awarded a fellowship at King's. In the spring of 1935, still only twenty-two years old, Turing was indeed elected as a Fellow of King's for three years, with the prospect of renewal for at least a further three years, at a stipend of £300 per year; the success was sufficiently remarkable that the boys at Sherborne were given a half-day holiday in his honour. But something much more significant had happened to Turing during his studentship year. He had been introduced to the puzzle of whether it was possible to establish, from some kind of mathematical first principles, whether or not a mathematical statement (such as Fermat's famous Last Theorem) was provable. Apart from the philosophical interest in the problem, if such a technique existed it would save mathematicians from wasting time trying to prove the unprovable.

A very simple example of an unprovable statement is 'this statement is false'. If it is true, then it must be false, and if it is false, it must be true. So it cannot be proved to be either true or false. The mathematical examples are more tricky, for those of us without a Part III in maths, but the principle is still the same. Embarrassingly for mathematicians, it turns out that there are mathematical statements which are true, but cannot be proved to be true, and the question is whether provable statements (equivalent to 'this statement is true') in mathematics can be distinguished from unprovable statements using some set of rules applied in a certain way.

Turing's introduction to these ideas came from a series of lectures given by Max Newman on 'The Foundations of Mathematics', drawing heavily on the work of the German mathematician David Hilbert. Newman described the application of this kind of set of rules as a 'mechanical process', meaning that it could be carried out by a human being (or a team of such human 'computers') following the rules blindly, without having any deep insight. As the Cambridge mathematician G. H. Hardy had commented, 'it is only the very unsophisticated outsider who imagines that mathematicians make discoveries by turning the handle of some miraculous machine'. But Turing, always idiosyncratic and literal-minded, saw that a 'mechanical process' carried out by a team of people *could* be carried out by a machine, in the everyday sense of the word. And he carried with him the idea, from his childhood reading, of even the human body as a kind of machine. In the early summer of 1935, as he lay in a meadow at Grantchester taking a rest from a long run, something clicked in his mind, and he decided to try to devise a machine that could test the provability of any mathematical statement. By then, he had already met von Neumann, who visited Cambridge in the spring of 1935, and had applied for a visiting fellowship at Princeton, von Neumann's base, for the following year. He would not arrive empty-handed.

Turing came up with the idea of a hypothetical automatic machine that would operate by reading and writing symbols on a long piece of paper – a paper tape. The tape would be divided into squares, and each square would either contain the symbol '1' or be blank, corresponding to the symbol '0'. The way in which the machine was set up would determine its initial 'state'. The tape would start off with a string of 1s and 0s, representing a problem that had to be solved – as Turing was well aware, any information can be represented in such a binary code, if the string of 1s and 0s is long enough.

This may strike you as odd, because the binary 'code' seems so simple. But the printed version of this book, for example, contains a certain amount of information 'stored' in the words of the English language and the letters of the alphabet. It could be translated into binary language simply by setting A = 0, B = 1, C = 10, D = 11 and so on, with extra binary numbers for punctuation marks, and writing out the string of 1s and 0s on a paper tape. Something similar, but not using this particular code, happens when my words are processed by the computer on which I write, at the printer's when the code is turned into printed pages, and, if you are reading an electronic version of the book, inside your reader.

The machine Turing described would, when setting out to solve a specific problem, read the first symbol on a tape, and, in accordance with its state at the time, either erase a 1, print a 1, or do nothing. Then it would move on to the next square, and act in accordance with its new state, which would have been determined by what happened at square 1. It could move forwards and backwards along the tape, but only one square at a time, writing and erasing symbols, until it reached a state corresponding to the end of its task. It would then stop, and the string of 1s and 0s left on the tape would represent the solution to the problem the machine had been working on. And it would have been done by a purely 'mechanical' process, owing nothing to inspiration or human intuition.

In terms of the original problem he had set out to solve, Hilbert's provability question, Turing's hypothetical machine was a great success. Simply by considering the way in which such a machine would work, he was able to show, using a detailed argument which we do not need to go into here, that there are uncomputable problems, and that there is no way to distinguish provable statements in mathematics from unprovable statements in mathematics using some set of rules applied in a certain way. This was impressive enough. But what is even more impressive, and the main reason why Turing's paper 'On Computable Numbers' is held in such awe today, is that he realized that his 'automatic machine' could be a universal computer. The way the machine works on a particular problem depends on its initial state. It is a limited machine that can only solve a single problem. But as Turing appreciated, the initial state can be set up by the machine reading a string of 1s and 0s from a tape – what we now call a computer program. The same piece of machinery (what we now call hardware) could be made to do any possible task in accordance with appropriate sets of instructions (what we now call software). One such machine can simulate the work of any such machine. And such machines are now known as Turing machines. In his own words, 'it is possible to invent a single machine which can be used to compute any computable sequence'.

The relevance of this idea to the logical puzzle that triggered Turing's investigation is that although he proved that it is possible to construct a machine to solve any solvable problem, it is not possible to construct a machine which can predict how many steps it will take to solve a particular problem. This is what establishes that although you can build an automaton to do anything that can be done, you cannot build a machine which tells you what can and can't be done. Logicians appreciate the full importance of this proof; but that is less important to us here than the fact that Turing machines exist.

A Turing machine simulates the activity of any specialist computer, using different sets of software. This is exactly what my iPhone, for example, does. It can be a phone, a TV or a navigation aid; it can play chess, solve certain kinds of mathematical problems, and do many other things. It can even do things its designers never thought of, as when an outside programmer devises a new app. Most people in the developed world now own, or have access to, a Turing machine, less than eighty years after the publication of 'On Computable Numbers'.

The paper was completed in the spring of 1936, just after the German army re-occupied the Rhineland, and it was published just under a year later, in the *Proceedings of the London Mathematical Society*. In the interim, an inconvenient blip occurred. Just a month after he had read an early draft of Turing's paper, Max Newman received a copy of a paper by Alonzo Church, a mathematician based at Princeton, in which he reached the same conclusion about Hilbert's question, using a technique he called lambda calculus. In a sense, Turing had been pre-empted, and although his version was still worth publishing, he had to add an appendix establishing that his work and Church's work were equivalent. Nobody realized, at the time, that the really important discovery described in that paper was the principle of the universal Turing machine.

. . . and Princeton

Encouraged by Newman and the possibility of working with Church, Turing was determined to make his visit to Princeton. He had applied for one of the Proctor Fellowships offered by Princeton; there were three of these each year, one for a